Name:_____   Teacher:_____

ALGEBRA 1 SUMMER ASSIGNMENT

Summer 2020

Dear Algebra 1 Students and Parents:

Welcome to Algebra 1! For the 2020-2021 school year, we would like to focus your attention to the prerequisite skills and concepts for Algebra 1. In order to be successful for Algebra 1, a student must demonstrate a proficiency in:

- ❖ The Number System
- ❖ Operations with Integers and Fractions
- ❖ Solving Equations

The attached review packet is provided for practice. Students are expected to have the packets completed by the first day of school. Teachers will review the answers during the first two or three days the class meets in September. As prerequisite skills, these topics are <u>not</u> re-taught in the Algebra 1 course. To ensure that all students demonstrate an understanding of the basic Algebra skills to be successful, these topics will be assessed as part of the Unit One Test in September. Students are encouraged to seek extra help before or after school from their teacher for any topics requiring more personal, in-depth remediation.

It is expected that each student will fully complete the review questions. If you have any questions, please do not hesitate to contact your child's teacher.

RESOURCES:

The Number System
https://www.purplemath.com/modules/numtypes.htm

Operations With Integers
https://www.katesmathlessons.com/adding-positive-and-negative-numbers.html

Operations with Fractions
https://www.katesmathlessons.com/adding-and-subtracting-fractions.html
https://www.katesmathlessons.com/multiplying-fractions.html
https://www.katesmathlessons.com/dividing-with-fractions.html

Evaluate Variable Expressions Using Substitution
https://www.katesmathlessons.com/order-of-operations.html

Combining Like Terms
https://www.katesmathlessons.com/adding-and-subtracting-polynomials.html

Solving Equations:

Two Step
https://www.katesmathlessons.com/solving-two-step-equations-p2.html

Variables on Both Sides
https://www.katesmathlessons.com/solving-equations-with-variables-on-both-sides.html

Special Equations – Solutions: $x = 0$, no solutions, infinite solutions (all Real Numbers)
https://www.purplemath.com/modules/solvelin5.htm

Proportions
https://www.katesmathlessons.com/solving-proportions.html

The Number System

IMAGINARY NUMBERS

Any number that does not have a place on the number line.

REAL NUMBERS

Any number that has a place on the number line.

IRRATIONAL NUMBERS

- Non-terminating, non-repeating decimals.
- Any number that cannot be expressed as a fraction.

RATIONAL NUMBERS

- Terminating or repeating decimals
- Fractions

INTEGERS

Whole numbers and their opposites.

WHOLE NUMBERS

Natural Numbers and Zero.

NATURAL NUMBERS

Counting Numbers.

Examples: Mark each box that described the given number.

Number	Natural	Whole	Integer	Rational	Irrational	Imaginary
7	✔	✔	✔	✔		
0		✔	✔	✔		
$2.\overline{37}$				✔		
$\sqrt{2}$					✔	
$\frac{2}{3}$				✔		
-3			✔	✔		
$\sqrt{-2}$						✔

Practice:

Number	Natural	Whole	Integer	Rational	Irrational	Imaginary
1) 25						
2) -14						
3) $\frac{5}{7}$						
4) 0.32						
5) -3.76						
6) $\sqrt{8}$						
7) 23						
8) $\sqrt{100}$						
9) -1.9						
10) 0.47892...						
11) $\sqrt{96}$						
12) $9.\overline{39}$						
13) $\sqrt{-9}$						
14) π						

OPERATIONS WITH INTEGERS

ADDITION:

1) **S**AME SIGNS → FIND THE **S**UM OF THE ABSOLUTE VALUE → KEEP THE SIGN

$$2 + 3 = 5 \qquad\qquad (-2) + (-3) = (-5)$$

2) **D**IFFERENT SIGNS → FIND THE **D**IFFERENCE OF THE ABSOLUTE VALUE → $|>|$
(USE THE SIGN OF THE NUMBER WITH
THE GREATER ABSOLUTE VALUE)

$$2 + (-3) = -1 \qquad\qquad (-2) + 3 = 1$$

SUBTRACTION: SUBTRACTION MEANS ADDING THE OPPOSITE:

$2 - 3 =$	$(-2) - 3 =$	$2 - (-3) =$	$(-2) - (-3) =$
$2 + (-3) = (-1)$	$(-2) + (-3) = (-5)$	$2 + (+3) = 5$	$(-2) + (+3) = 1$

MULTIPLICATION & DIVISION:

LIKE SIGNS → POSITIVE PRODUCT

$$2 \cdot 5 = 10 \qquad\qquad (-2) \cdot (-5) = 10$$

UNLIKE SIGNS → NEGATIVE PRODUCT

$$(-2) \cdot 5 = (-10) \qquad\qquad 2 \cdot (-5) = (-10)$$

OR…COUNT THE NUMBER OF NEGATIVE SIGNS:

EVEN # → POSITIVE PRODUCT $\qquad (-2) \cdot 5 \cdot (-1) \cdot (-1) \cdot (-1) = 10 \qquad\qquad$ (4 negative signs)

ODD # → NEGATIVE PRODUCT $\qquad (-2) \cdot 5 \cdot (-1) \cdot (-1) \cdot (-1) \cdot (-1) = (-10) \qquad$ (5 negative signs)

FOO – FUNDAMENTAL ORDER OF OPERATIONS

1) GROUPING SYMBOLS:

Parentheses ( )	Brackets [ ]
Braces { }	Absolute Value Symbol $\lvert\;\rvert$
Fraction Bar	Radical Sign $\sqrt{}$

2) EXPONENTS:

3) MULTIPLICATION & DIVISION: LEFT TO RIGHT (FIRST COME, FIRST SERVE)

4) ADDITION & SUBTRACTION: LEFT TO RIGHT (FIRST COME, FIRST SERVE)

PRACTICE: Simplify each expression. All answers should be written in simplest form.

1) $-2 - 3 - 4$

2) $-5 - 6 + 7$

3) $3 - 4 + 4$

4) $3 - 4 + 5 - 6$

5) $9 - 7 + 5 - (-3)$

6) $-7 - (-7) - 7 - 2 + (-3)$

7) $\left[-7 \cdot 8 + (-3)\right] + 4^2$

8) $4(-2)^5 - 21 \div (-7)$

9) $\left\{(-5 + 1) - \left[-24 \div (-2)\right]\right\}^2$

10) $-12 - 57 \div 3 \div (-1)^9$

OPERATIONS WITH FRACTIONS

Adding Fractions: To add or subtract fractions, you must have a *common denominator*.

$$\frac{1}{3} + \frac{3}{4} \quad \rightarrow \quad \frac{1}{3}\left(\frac{4}{4}\right) + \frac{3}{4}\left(\frac{3}{3}\right) \quad \rightarrow \quad \frac{4}{12} + \frac{9}{12} \quad \rightarrow \quad \frac{13}{12}$$

Multiplying Fractions: To multiply fractions, cancel when possible, multiply the *numerator* and the *denominator*, then *simplify*.

$$\left(-\frac{2}{7}\right)\left(\frac{1}{4}\right) \quad \rightarrow \quad (-1)\left(\frac{2 \cdot 1}{7 \cdot 4}\right) \quad \rightarrow \quad \left(-\frac{2}{28}\right) \quad \rightarrow \quad \left(-\frac{1}{14}\right)$$

Dividing Fractions: To divide fractions, multiply the first term by the **RECIPROCAL** of the second term.

$$\left(\frac{1}{2}\right) \div \left(\frac{3}{4}\right) \quad \rightarrow \quad \left(\frac{1}{2}\right) \cdot \left(\frac{4}{3}\right)^{2} \quad \rightarrow \quad \left(\frac{2}{3}\right)$$

PRACTICE: Simplify each expression. All answers should be written in simplest form.

Do not convert improper fractions to mixed numbers!

1) $\quad -\dfrac{4}{3} + \dfrac{9}{5}$

2) $\quad -\dfrac{4}{5} - \dfrac{5}{8}$

3) $\quad \left(-\dfrac{4}{3}\right)\left(-\dfrac{3}{5}\right)$

4) $\quad \left(\dfrac{10}{7}\right)\left(-\dfrac{1}{6}\right)$

5) $\quad \left(-\dfrac{4}{3}\right) \div \left(\dfrac{3}{5}\right)$

6) $\quad \dfrac{1}{2} + \left(-\dfrac{3}{8}\right)$

7) $\quad -\dfrac{1}{2} - \left(-\dfrac{3}{8}\right)$

8) $\quad \left(\dfrac{2}{7}\right) \div \left(\dfrac{3}{2}\right)$

EVALUATE VARIABLE EXPRESSIONS USING SUBSTITUTION

1)   Substitute the given values for each variable.
2)   Use PARENTHESES when substituting negative numbers.
3)   Simplify using FOO.

Evaluate each expression if a = 4, b = −1, c = $\frac{1}{4}$, and d = $\frac{1}{2}$. Use parentheses when plugging values in!

1) $2a^2 - 3b + 4 =$

2) $-3b^3 - 4c + 2d =$

3) $\dfrac{ad + d}{d^2} =$

4) $\dfrac{a + d}{ac - b} =$

5) $\dfrac{abc}{d} =$

6) $-b^3 + a^2 - d =$

COMBINING LIKE TERMS

Expression: a mathematical phrase that contains operations, numbers and/or variables.

Term: a number, a variable or a product or quotient of numbers or variables that is added or subtracted in an algebraic expression.

- *There are 4 terms in the following expression*: $2x - 4y + 7z + 3$

Variable: a symbol (usually a letter) used to represent a quantity that can change.

Coefficient: a number that is multiplied by a variable.

- *In the term* $2x$, 2 *is the coefficient. This means* 2 *times the quantity x.*
- *In the term x, the coefficient is understood to be* 1, *even though the number is not written.*

Constant: a term in an algebraic expression that does not change; it does not contain variables.

- *In the expression x + 2, 2 is a constant.*

Like Term: a term that has the <u>same</u> variable (letter) raised to the <u>same</u> power.

Equivalent: having the same value.

Equation: a mathematical sentence that shows that two expressions are equivalent.

- *The expression* $2x + 7x + 3 - 2$ *can be written as an equivalent expression* $9x + 1$ *after combining like terms.*
- *The expression* $2x - 4y + 7z + 3$ *cannot be simplified because none of the terms are like terms*.

ADDITION AND SUBTRACTION: ONLY LIKE TERMS CAN BE COMBINED.
　　　　1) Distribute, if necessary.
　　　　2) Combine like terms by adding or subtracting the coefficients of all like terms.

 Examples:

 a) $3x + 2x = 5x$　　　　b) $3x + 2y$ (CANNOT BE SIMPLIFIED)　　　　c) $3x + 10 + 3y + 2x - 2y + 13 = 5x + y + 23$

PRACTICE: Simplify each expression. All answers should be written in simplest form.

1) $5x + 6xy - 7x + 8yx$

2) $3x - 4(2x - 2) - 5$

3) $2(6x + 15) + 3(4x - 10)$

4) $8x - 3x + 9x^2 - 5x + 2x$

5) $2 - (5x - 10) + 7 - x$

6) $(x + 1) - (2x + 3) - (4x - 5)$

SOLVING EQUATIONS

A mathematical sentence with one or more variables is called an **open sentence**. Open sentences are **solved** by finding replacements for the variables that result in true sentences. A sentence that contains an equal sign, =, is called an **equation**.

To solve an equation, find the value or values of a variable that make the equation true. To do this, use inverse operations to **ISOLATE THE VARIABLE**. In other words, get the variable on one side of the equal sign with a coefficient of 1. The Properties of Equality enable us to use inverse operations to isolate the variable and thus solve an equation.

STEPS FOR SOLVING EQUATIONS

1) **Distribute**, if necessary.
2) **Combine Like Terms**, if necessary.
3) **Move the variables** to one side of the equal sign. (Keep coefficient positive, if possible.)
4) **Isolate the variable**. Perform inverse operations to move the constants to the other side of the equation.
5) **Divide by the coefficient** (if the coefficient ≠ 1)

PRACTICE: Solve for x. *Express all answers in simplest form.* *Do not convert improper fraction to mixed numbers!*

1) $\dfrac{1}{2}x = 10$

2) $4x = -\dfrac{1}{4}$

3) $10 - x = -10$

4) $-4 - \dfrac{x}{3} = 1$

5) $7 - 3x = -17$

6) $-\dfrac{2}{3}x + 5 = 13$

7) $\dfrac{x}{-7} + 2 = 0$

8) $5(4x - 3) = 3$

9) $-5 + 7x = 16$

10) $10x + 1 = 15x - 9$

11) $-(x - 5) - 5 = 3(3x - 10)$

12) $2(3x - 2) - 5 = 2(2x - 4)$

13) $32 + 2(x - 2) = -3(2x + 4)$

14) $8x - (1 - x) = 7x - 1$

15) $4x + 2(3x - 3) = 4 - 4(2x + 1)$

SPECIAL EQUATIONS (*to be reviewed)

Sometimes when you solve an equation, the variables are eliminated from the entire equation and you end up with a statement that can be true or false. If the statement is true, then the solution set = $\Re$. An equation that always produces a true result is called an *identity*. If the statement is false, then the solutions set = $\varnothing$. An equation that always produces a false result is called a *contradiction*.

IDENTITY: When solving the equation, the resulting statement is TRUE. $\rightarrow$ $x = \Re$

$$6x + 9 = 6x + 9$$
$$\underline{-6x \qquad -6x}$$
$$9 = 9$$

$\rightarrow$ TRUE $\rightarrow$ x = The Set of Real Numbers $\rightarrow$ $x = \Re$

CONTRADICTION: When solving the equation, the resulting statement is FALSE. $\rightarrow$ $x = \varnothing$

$$2(3x + 4) = 6x - 5$$
$$6x + 8 = 6x - 5$$
$$\underline{-6x \qquad -6x}$$
$$8 = 5$$

$\rightarrow$ FALSE $\rightarrow$ x = The Null Set $\rightarrow$ $x = \varnothing$

PRACTICE: Solve for x. *Express all answers in simplest form.* *Do not convert improper fractions to mixed numbers!*

1) $3(x - 2) = 3x - 6$

2) $3x + 5 = 5 - 3x$

3) $17 + 8x = 2(4x + 8)$

4) $x + 2 = x$

5) $-x + 3 = -2 - 4x$

6) $3x = 8x - 5x$

PROPORTIONS

A *proportion* is an equation that states that two ratios are equivalent. In a proportion, the cross products are equal. If one number in a proportion is unknown, you can find the missing number by finding the cross products and solving the equation. If there is a sum or a difference in a numerator or denominator, use parentheses before you find the cross products (example c).

Examples:

a) $\dfrac{2}{3} = \dfrac{5}{15}$ → $2 \cdot 15 = 3 \cdot 5$ → $15 = 15$

b) $\dfrac{2}{3} = \dfrac{5}{x}$ → $2x = 3 \cdot 5$ → $2x = 15$ → $x = \dfrac{7}{2}$

c) $\dfrac{(2x + 3)}{3} = \dfrac{5}{2}$ → $2(2x + 3) = 15$ → $4x + 6 = 15$ → $4x = 9$ → $x = \dfrac{9}{4}$

d) $\dfrac{2x}{3} = 24$ → $\dfrac{2x}{3} = \dfrac{24}{1}$ → $2x = 72$ → $x = 36$

PRACTICE: Solve for x. Express all answers in simplest form. Do not convert improper fraction to mixed numbers!

1) $\dfrac{2}{x} = \dfrac{4}{15}$

2) $\dfrac{2x}{3} = 5$

3) $\dfrac{4x - 3}{9} = 3$

4) $\dfrac{x - 2}{2} = \dfrac{x}{3}$

5) $\dfrac{x}{7} = \dfrac{x + 4}{3}$

6) $\dfrac{2x - 5}{6} = \dfrac{3x - 1}{3}$